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Risk-based determination of heat demand for central and district heating by a probability theory approach

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ABSTRACT

One of the most important design tasks related to the establishment and operation of heat supply systems involve design heat demand of a required reliability level. Determination of heat demand for central and district heating is set out in standards in the practice of both Hungary and most European countries. Methodologies reflect a deterministic tendency. The uncertainty of heat demand for heating so specified is not analyzed: values are not associated with reliability, probability and confidence interval figures. Our article presents the method of risk-based determination of heat demand for central and district heating by probability theory approach.

1. Introduction

One of the most important design tasks related to the establishment and operation of heat supply systems involve design heat demand of a required reliability level. Heat demand can be for heating and for domestic hot water.

It is a fact in heat supply that both heat demand for central and district heating are random, meaning that they depend on a number of factors, and they cannot be forecasted accurately. They can be specified primarily by a probability approach [1].

In terms of tendency and volume, heat demand for heating are linked to the magnitude and frequency of occurrence of external meteorological factors, especially external temperature and winds. Demand for domestic hot water change absolutely randomly: their size depends, in the stochastic sense, on the number of homes supplied and the number of inhabitant consumers.

Brochus et al. [3] presented the quantification of uncertainty in predicting building energy consumption with stochastic approaches to building energy use. A probabilistic approach for the thermal performance of a building envelope was published by Pietzyk [4]. Wang et al. [6] investigated the uncertainties in energy consumption introduced by building operations and weather for medium-size office building. Nagai and Nagata [7] presented the probabilistic approach to determination of internal heat gains in office buildings for peak load calculation. Pedersen et al. [8] presented the load prediction method for heat and electricity demand in buildings for the purpose of planning for mixed energy distribution systems. Hovgaard et al. [11] investigated model predictive control technologies in refrigeration systems. Rezvan et al. [12] presented the optimization of distributed generation capacities in buildings of uncertainty in load demand for combined heat and power units. A research group in Sweden [13,14] investigated a multi-agent approach to deliver hot water just in time to the district heating system. Simulation models have also been developed [5]. However, specialists have not yet dealt with the risk-based determination of heat demand.

Design heat demand means a maximum of heat demand which occurs at a less than 1% frequency rate (lasting for 24 h) in the coldest appreciable external weather conditions (between −13 and −15 °C).

A maximum design value is also characteristic of demand for domestic hot water. It is required to be determined in order to specify production capacity volumes. This issue was discussed in [2]. This paper analyzes the uncertainties of heat demand for central and district heating by applying a mathematical probability theory. Uncertainty can be examined in terms of the heat demand of an apartment or a building, or the total heat demand of a district heating system. Uncertainty of the total system can be built up from the heat demand uncertainties of apartments and buildings (expected value, standard deviation and confidence interval). This has been termed as the synthetic method. The uncertainty of

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the resultant heat demand of a system can also be determined from measured values by using correlation and regression analysis tools. Both methods are presented in our study.

It can be stated on this basis to what extent it is likely that actual heat demand will not exceed a prescribed value at a certain time of day, a function of the external meteorological condition, and taking a certain number of homes as a basis.

Even in daily operational practice, it is very important to know the rate of heat demand expected to occur, and accordingly, the schedule to be applied as well as the forward water temperature and mass flux to be planned.

The method outlined here is used for finding new ways and means to specify various demands in building engineering design, hoping that oversized building engineering systems – including their investment and operating costs – can be reduced. So far, our investigations suggest that 10–15% capacity reserves can be found by these methods compared to the results of calculations performed according to a deterministic approach.

The following is a presentation of the theory of this analysis.

2. Probability character of heat demand for central and district heating in case of an apartment or a building

First of all, let us examine the determination of apartment and building heat demand and the arising uncertainty in a probability field. As mentioned in the introduction, the uncertainty of the heat demand of apartments or buildings can be used for building up the uncertainty of the heat demand of the entire district heating system synthetically. Heating aims to ensure agreeable comfort parameters for those who are staying within a space of residence, primarily pleasant air temperatures (based on [9], 20–24 °C pertaining to category II) on an on-going basis.

Heat demand for heating for an apartment:

$$\dot{Q} = \sum U \cdot A \cdot \Delta t + \sum \dot{V} \cdot \rho \cdot c \cdot \Delta t + \sum \dot{Q}_b - \sum \dot{Q}_k, \quad (1)$$

where U is the heat transmission coefficient; A is the delimiting surface; $\dot{V}$ is the volumetric flow of filtration; Δt is the difference between internal and external temperatures where the internal temperatures is a prescribed or a given value set by the consumer; $\dot{Q}_b$ means the heat output of internal heat sources, internal heat gain, and the heat output of household machinery and people; $\dot{Q}_k$ means external heat sources (solar radiation); i number of apartments.

For one building, formula (1) is used for summarizing the heat demand determined for each apartment accordingly.

In this context, all factors are probability type: they cannot be specified exactly and they change randomly. As a consequence, heat demand $\dot{Q}$ for heating is a probability type factor, a probability variable in the mathematical sense. Heat gain from solar radiation and from heat produced within the building is also random. Therefore, heat demand for heating involves uncertainties and has a probability nature even in case of given external meteorological conditions.

A probability variable is determined by the type of probability distribution, distribution parameters, the expected value and standard deviation.

In the course of design, it cannot be forecasted accurately the actual values of U, A and V cannot be forecasted accurately as opposed to plans, and it cannot be stated what values are assumed by U and $\dot{V}$ in case of equipment in operation in certain weather conditions. The figures realized are unknown and uncertain compared to design values, meaning that they represent probability variables. It should also be added that the values of heat transfer and heat transmission coefficients affecting heat transmittance – as set out in manuals – similarly involve uncertainties, so they also constitute probability variables. Obviously, internal temperatures are also within a certain range: their values are set by consumers, so they are also of the probability type.

Determination of the so-called standard (nominal) heat demand is set out in the currently effective standard or regulation of heat demand design. The calculations and dimensioning procedures described in the standard are deterministic. Designers are required to analyze and calculate the uncertainty and the probability distribution of figures calculated according to standard specifications.

2.1. Devise of the design (nominal, maximum) heat demand for heating

In the design of a heated object, the design heat demand for heating is specified as follows:

Assignment: to seek for the maximum of expression (1) at a prescribed reliability level:

$$\dot{Q}_n = \max_{\Delta t} \left\{ \dot{Q} = A \cdot U \cdot \Delta t + \dot{V} \cdot \rho \cdot c \cdot \Delta t - \dot{Q}_b - \dot{Q}_k \right\},$$
$$R = P\left(\dot{Q} \geq \dot{Q}_n\right) \leq 0.01, \quad (2)$$

where R is the chance of occurrence of a heat demand exceeding the design heat demand (representing the risk).

Theoretically speaking, all the variables in the equation are known design values, but the states actually realized are of the probability type.

Based on the statistics of local meteorological factors, the external design temperature is usually −10 to −15 °C.

In the expression above, R designates the rate of risk, specified at 1% in general for design in practice.

The equation must be solved for each of the values −10 to −15 °C, and the expected value and standard deviation of the rest of the factors are analyzed for each value assumed. It can be presumed that standardized, the so-called ‘required heat’ calculations yield the expected value of heat demand. Our task is to analyze the uncertainties of the influencing factors $U, A, \dot{V}, \dot{Q}_b, \dot{Q}_k$.

Distribution of the function $\dot{Q}$ can be determined partly by calculations and partly by measurements on basis of the distribution of variables. The method for this is presented below. Correlation (2) includes multiplications and an addition; accordingly, it should be investigated how to calculate the distribution parameters of the product and sum of probability variables.

2.2. Sum of two probability variables

2.2.1. Density and distribution functions

In order to determine the probability distribution of the sum of two probability variables, let us take probability variables x_1, x_2 of joint density function $h(x_1,x_2)$ and probability variable $y = x_1 + x_2$.

If the joint density function of probability variables x_1, x_2 is $h(x_1,x_2)$, then the density function of their sum $y = x_1 + x_2$ will be

$$g(y) = \int_{-\infty}^{\infty} h(x, y-x)dx = \int_{-\infty}^{\infty} h(y-x, x)dx. \quad (3)$$

In view of the density function of the sum, let us calculate the distribution function of $y = x_1 + x_2$ as well.

$$G(y) = \int_{-\infty}^{y} \int_{-\infty}^{\infty} h(x, z-x)dxdz = \int_{-\infty}^{y} \int_{-\infty}^{\infty} h(z-x, x)dxdz. \quad (4)$$

If x_1 and x_2 are independent probability variables, $h(x_1,x_2) = f_1(x_1)f_2(x_2)$ (where $f_1(x_1)$, and $f_2(x_2)$ are the density functions of x_1, and x_2 respectively), then the density function of their sum will be as follows, based on (3):

$$g(y) = \int_{-\infty}^{\infty} f_1(x) f_2(y-x) dx = \int_{-\infty}^{\infty} f_1(y-x) f_2(x) dx \quad (5)$$

and taking into consideration the distribution function of their sum, (5):

$$g(y) = \int_{-\infty}^{\infty} F_2(y-x) f_1(x) dx = \int_{-\infty}^{\infty} F_1(y-x) f_2(x) dx \quad (6)$$

where it is taken into consideration that

$$\int_{-\infty}^{y} f_2(z-x) dz = F_2(y-x) \quad \text{and} \quad \int_{-\infty}^{y} f_1(z-x) dz = F_1(y-x) \quad (7)$$

is the distribution function of x_1, and x_2 respectively.

2.2.2. Expected value and standard deviation of the sum and the difference of independent probability variables

Standard deviation of a probability variable:

$$D(x) = (\sigma(\xi)) = +\sqrt{M((x-M(x))^2)} \quad (8)$$

Variance:

$$D^2(x) = M(x^2) - (M(x))^2. \quad (9)$$

If $y = ax + b$, and a and b are constant, then

$$M(y) = aM(x) + b \quad (10)$$

$$D^2(y) = a^2 D^2(x). \quad (11)$$

In case of independent probability variables, the expected value of the sum of arbitrary probability variables x and y equals to the sum of the expected values of the members, that is, if $M\{x\}$ and $M\{y\}$ exist, then $M\{x+y\}$ also exists and

$$M\{x+y\} = M\{x\} + M\{y\}. \quad (12)$$

So the expected value of a sum equals to the sum of the expected values of its members, that is,

$$M\{x_1 + x_2 + \cdots + x_n\} = M\{x_1\} + M\{x_2\} + \cdots + M\{x_n\}. \quad (13)$$

By further narrowing the scope of the variables examined to those of normal distribution, it can be stated that if x_1 is an independent probability variable of $N(m_1, \sigma_1)$ distribution, and x_2 is an independent probability variable of $N(m_2, \sigma_2)$ distribution, then their sum $x_1 + x_2$ will also be of normal distribution with the parameters $m = m_1 + m_2$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2$. And for the case of more than two variables,

$$m = m_1 + m_2 + \cdots + m_n, \quad (14)$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2, \quad (15)$$

where m stands for the expected value, and σ for standard deviation.

If x and y are independent, then it holds true for variance that

$$D^2(x+y) = D^2(x-y). \quad (16)$$

If $x_1, x_2, \ldots, x_n$ are independent probability variables of identical distribution, then for their sum, that is, for probability variable

$$\xi_n = x_1 + x_2 + \cdots + x_n \quad (17)$$

$$M(\xi_n) = nM, \quad (18)$$

and

$$D(\xi_n) = \sqrt{n} D. \quad (19)$$

Relative standard distribution:

$$D_R(\xi_n) = \frac{D}{M(\xi_n)} = \frac{D}{\sqrt{n} M}. \quad (20)$$

2.3. Product of two probability variables

2.3.1. Density and distribution functions

In order to determine the density function of the product of two probability variables, let us take (x, y) two-dimensional probability vector variable of density function $h(x,y)$, and calculate its density function $z = xy$.

The density function of $z = xy$, to be marked by $h(z)$

$$h(z) = \int_{-\infty}^{\infty} g(w, z) dw = \int_{-\infty}^{\infty} h\left(w, \frac{z}{w}\right) \frac{1}{|w|} dw, \quad (21)$$

and $z = y$, $w = x$, $h(y) = g(y)$ by transcription (and for symmetry reasons)

$$g(y) = \int_{-\infty}^{\infty} h\left(\frac{y}{x}, x\right) \frac{dx}{|x|} = \int_{-\infty}^{\infty} h\left(x, \frac{y}{x}\right) \frac{dx}{|x|}. \quad (22)$$

If x_1 and x_2 are independent, taking density functions $f_1(x_1)$ and $f_2(x_2)$, then the density function of their product will be as follows, considering (21):

$$g(y) = \int_{-\infty}^{\infty} f_1\left(\frac{y}{x}\right) f_2(x) \frac{dx}{|x|} = \int_{-\infty}^{\infty} f_1(x) f_2\left(\frac{y}{x}\right) \frac{dx}{|x|} \quad (23)$$

In respect of independent probability variables, it can be established that if x and y are independent probability variables, then their product will equal to the product of their expected value, that is,

$$M\{xy\} = M\{x\} M\{y\}. \quad (24)$$

Our proposition also applies to the product of an arbitrary finite number of probability variables; that is, if $x_1, x_2, \ldots, x_n$ are independent probability variables, then

$$M\{x_1, x_2, \ldots, x_n\} = M\{x_1\} M\{x_2\} \cdots M\{x_n\}.$$

Another very important statement can be made regarding the standard deviation (D) of the product of independent probability variables, according to which if x_1 and x_2 are independent probability variables, then

$$D^2(x_1 x_2) = D^2(x_1) D^2(x_2) + M^2(x_1) D^2(x_2) + M^2(x_2) D^2(x_1). \quad (25)$$

2.3.2. Standardized probability variable

If ζ is a probability variable with an expected value $M(\zeta)$ and standard deviation $D(\zeta)$, then for the standardized probability variable

$$\xi = \frac{\zeta - M(\zeta)}{D(\zeta)}$$

$$M(\xi) = 0 \quad \text{and} \quad D(\xi) = 1. \quad (26)$$

2.3.3. Probability variable of normal distribution

Density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad -\infty < x < \infty. \quad (27)$$

Distribution function

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-m)^2}{2\sigma^2}} dt. \quad (28)$$

In case of the standardized normal distribution probability variable, $m = 0$ and $\sigma = 1$.

Density function and distribution function values can be looked up in technical books and tables.

3. Determination of the heat demand uncertainty by the probability distribution of an apartment and/or a building

The uncertainty of a consumer's heat demand can be established from the uncertainties of the factors in expression (1). Expression (21) can be used for determining the standard deviation of the product of the heat transfer coefficient and the cooling surface – if the expected values of these factors are known. The uncertainty of internal and external heat gain can be analyzed separately, the same as the volumetric flow of filtration.

In view of the expected value and standard deviation of each member of expression (2), the expected value of heat demand for heating can be specified by expression (13), and its standard deviation by expression (15).

Based on the above, the statistical parameters of heat demand for convection heating are:

$$D^2(UA\Delta t) = D^2(UA)D^2(\Delta t) + M^2(UA)D^2(\Delta t) + M^2(\Delta t)D^2(UA) \tag{29}$$

As Δt = constant if a well-known meteorological state is taken into consideration, therefore

$$M^2(\Delta t) = \Delta t^2, \tag{30}$$

$$D^2(\Delta t) = 0. \tag{31}$$

Using these,

$$D^2(UA\Delta t) = D^2(UA)\Delta t^2, \tag{32}$$

and based on (25)

$$D^2(UA) = D^2(U)D^2(A) + M^2(U)D^2(A) + M^2(A)D^2(U). \tag{33}$$

Expected value of heat demand

Expected value of (1) is

$$M(\dot{Q}_f) = M(UA)\Delta t + M(\dot{V}\rho c)\Delta t - M(\dot{Q}_b) - \dot{M}(\dot{Q}_k). \tag{34}$$

Standard deviation of heat demand Using the following correlation according to (16):

$$D^2(x+y) = D^2(x-y),$$

$$D^2(\dot{Q}_f) = D^2(UA)\Delta t^2 + D^2(\dot{V}\rho c)\Delta t^2 + D^2(\dot{Q}_b) + D^2(\dot{Q}_k). \tag{35}$$

Introducing the standard normal distribution probability variable of heat demand for heating,

$$\xi_f = \frac{\dot{Q}_f - m_f}{\sigma_f}.$$

From this, the heat demand as a probability variable is

$$\dot{Q}_f = m_f + \xi\sigma_f. \tag{36}$$

For instance, let us choose 99% for supply safety, meaning that the heat demand realized is required to be less or equal to the design value specified by us at 99% certainty. Then, on the basis of the table of the distribution, for which $M(\phi) = 0$, $\sigma(\phi) = 1$ it can be stated that where $P(\xi) = 0.99$, there $\xi = 2.33$.

Thereby the design value of heat demand, for which

$$P(\dot{Q} \geq \dot{Q}_{mf}) \leq 0.01,$$

$$\dot{Q}_{mf} = m_f + 2.33\sigma_f. \tag{37}$$

3.1. Analysis of the expected value and standard deviation of heat demand for an apartment and/or a building

The following is an example to illustrate the uncertainty and standard deviation of convection and filtration heat demand, internal and external heat gain, and the resultant heat required in case of various external temperatures and different values of uncertainty.

Relative standard deviation of heat gain:

$$D_R\left(\dot{Q}_b + \dot{Q}_k\right) = \frac{D\left(\dot{Q}_b + \dot{Q}_k\right)}{M\left(\dot{Q}_b + \dot{Q}_k\right)}. \tag{38}$$

Relative standard deviation of filtration heat loss:

$$D_R(\dot{V}\rho c) = \frac{D(\dot{V}\rho c)}{M(\dot{V}\rho c)}. \tag{39}$$

Relative standard deviation of specific transmission heat loss:

$$D_R(UA) = \frac{D(UA)}{M(UA)}. \tag{40}$$

Expected component values in convection heat loss calculations:

$$M(U) = U_0, \tag{41}$$

$$M(A) = A_0. \tag{42}$$

The standard deviations:

$$D(U) = b_1 U_0, \quad \text{where} \quad b_1 = \frac{D(U)}{M(U)}, \tag{43}$$

$$D(A) = b_2 A_0, \quad \text{where} \quad b_2 = \frac{D(A)}{M(A)}. \tag{44}$$

Then, based on (25) and (33), the variance of the required specific convection heat projected to a unit of internal and external temperature difference will be:

$$D^2(UA) = b_1^2 U_0^2 b_2^2 A_0^2 + b_2^2 U_0^2 A_0^2 + b_1^2 U_0^2 A_0^2 = \left(b_1^2 b_2^2 + b_1^2 + b_2^2\right) U_0^2 A_0^2, \tag{45}$$

the expected value:

$$M(UA) = U_0 A_0. \tag{46}$$

3.2. An example for the heat demand uncertainty of an apartment

In order to demonstrate the method deduced, the uncertainty of the heat demand of a typical Hungarian apartment is analyzed, which is provided with central heating and district heating supply. The standard heat demand of the apartment is 3700 W for sizing status, which is considered to be an expected value. For a selected numerical value, if $M(\dot{Q}_b + \dot{Q}_k) = 500$ W, $D(\dot{Q}_b + \dot{Q}_k) = 250$ W, $M(\dot{V}\rho c) = 60$ W/K, $D(\dot{V}\rho c) = 30$ W/K, $M(\text{kA}) = 60$ W/K and $D^2(\text{kA}) = 18.9737$ and 8.48 W^2/K^2 respectively, – formulas (32) and (34) were used to define the expected value $M(\dot{Q}_f)$ and standard deviation $D(\dot{Q}_f)$ of the heat required for heating ($\dot{Q}_f$) a function of external temperature.

Table 1 shows various expected values $M(UA)$ and relative standard deviations $D_R^2(UA)$ with the corresponding variance $D^2(UA)$ figures.

In Table 2, the standard deviations of specific filtration heat demand $D(\dot{V}\rho c)$ were shown for various expected values of specific filtration heat demand $M(\dot{V}\rho c)$ and its relative standard deviation $D_R(\dot{V}\rho c) = \frac{D(\dot{V}\rho c)}{M(\dot{V}\rho c)}$.

Table 1
Standard deviation of specific convection heat flux D(kA) as a function of the expected value $M(UA)$ and relative standard deviation.

$M(UA) = U_0A_0$ [W/K]	$(b_1^2 b_2^2 + b_1^2 + b_2^2) = D_R^2(UA) = D^2(UA)/M^2(UA)$ [−]							
	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16
10	1.41421	2	2.44949	2.82843	3.16228	3.46410	3.74166	4
20	2.82843	4	4.89898	5.65685	6.32456	6.92820	7.48331	8
30	4.24264	6	7.34847	8.48528	9.48683	10.39230	11.22500	12
40	5.65685	8	9.79796	11.31370	12.64910	13.85640	14.96660	16
50	7.07107	10	12.24740	14.14210	15.81140	17.32050	18.70830	20
60	8.48528	12	14.69690	16.97060	18.97370	20.78460	22.44990	24
70	9.89949	14	17.14640	19.79900	22.13590	24.24870	26.19160	28
80	11.31370	16	19.59590	22.62740	25.29820	27.71280	29.93330	32
90	12.72790	18	22.04540	25.45580	28.46050	31.17690	33.67490	36
100	14.14210	20	24.49490	28.28430	31.62280	34.64100	37.41660	40

Table 2
Standard deviation of specific filtration heat demand as a function of the expected value and relative standard deviation.

$M(\dot{V}\rho c)$ [W/K]	$D_R(\dot{V}\rho c) = D(\dot{V}\rho c)/M(\dot{V}\rho c)$ [−]							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
6	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8
12	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6
18	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4
24	2.4	4.8	7.2	9.6	12.0	14.4	16.8	19.2
30	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0
36	3.6	7.2	10.8	14.4	18.0	21.6	25.2	28.8
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0

In Table 3, the expected value $M(\dot{Q}_b + \dot{Q}_k)$ and relative standard deviation

$$D_k\left(\dot{Q}_b + \dot{Q}_k\right) = \frac{D\left(\dot{Q}_b + \dot{Q}_k\right)}{M\left(\dot{Q}_b + \dot{Q}_k\right)} \tag{47}$$

of internal and external heat gain are associated with the standard deviation of internal and external heat gain $[D(\dot{Q}_b + \dot{Q}_k)]$.

The values are shown in Fig. 1.

The values of the example in Fig. 1 clearly suggest the benefits of a probability approach as opposed to the deterministic approach; thus, the uncertainty to be reckoned with when planning heating operation schedules can now be measured.

4. Defining simultaneous heat demand for a group of consumers

If the heat demand of an n number of heating consumers is known one by one in a way that their distribution is known in terms of expected value (m) and standard deviation (σ), then our task is to define the design value

$$\dot{Q}_f = \dot{Q}_{1,f} + \dot{Q}_{2,f} + \cdots + \dot{Q}_{n,f} \tag{48}$$

of aggregate heat demand.

Assuming that all of $\dot{Q}_{1,f}, \dot{Q}_{2,f}, \ldots, \dot{Q}_{n,f}$ show normal distribution, let us state the variables of standard normal deviation derived from them:

$$\xi_1 = \frac{\dot{Q}_{1,f} - m_{1,f}}{\sigma_{1,f}}, \qquad \xi_2 = \frac{\dot{Q}_{2,f} - m_{2,f}}{\sigma_{2,f}}, \quad \ldots, \quad \xi_n = \frac{\dot{Q}_{n,f} - m_{n,f}}{\sigma_{n,f}}, \tag{49}$$

where m_{if} is the expected value of the heat demand of a single consumer,σ_{if} is the standard deviation of the heat demand of a single consumer.

From these expressions, the design values of consumer heat demand $\dot{Q}_{1,f}, \dot{Q}_{2,f}, \ldots, \dot{Q}_{n,f}$ as probability variables are:

$$\dot{Q}_{m1,f} = m_{1,f} + \xi_1\sigma_{1,f}, \qquad \dot{Q}_{m2,f} = m_{2,f} + \xi_2\sigma_{2,f}, \quad \ldots, \quad \dot{Q}_{mn,f} = m_{n,f} + \xi_n\sigma_{n,f}. \tag{50}$$

Table 3
Standard deviation of internal and external heat gain as a function of the expected value and relative standard deviation.

$M(\dot{Q}_b + \dot{Q}_k)$ [W]	$D_R(\dot{Q}_b + \dot{Q}_k) = D(\dot{Q}_b + \dot{Q}_k)/M(\dot{Q}_b + \dot{Q}_k)$ [−]							
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
100	10	20	30	40	50	60	70	80
200	20	40	60	80	100	120	140	160
300	30	60	90	120	150	180	210	240
400	40	80	120	160	200	240	280	320
500	50	100	150	200	250	300	350	400
600	60	120	180	240	300	360	420	480
700	70	140	210	280	350	420	490	560
800	80	160	240	320	400	480	560	640
900	90	180	270	360	450	540	630	720
1000	100	200	300	400	500	600	700	800

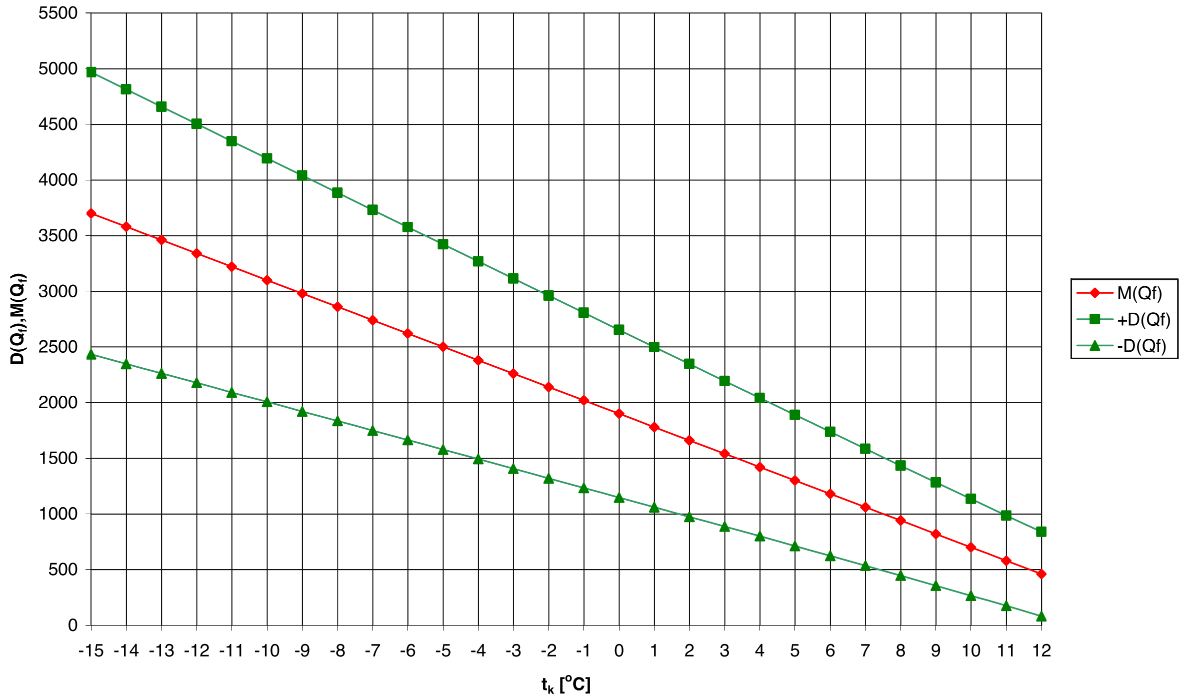

Fig. 1. Expected values and standard deviation of heat demand for heating a function of external temperature for a typical Hungarian flat in district heating.

Let it be $\xi_1 \equiv \xi_2 \equiv \cdots \equiv \xi_n \equiv \xi$.

Using correlations (14) and (15), the following can be stated for the calculation of simultaneous heat demand of a given reliability level in case of n heating consumers:

$$\dot{Q}_{m,f} = (m_{1,f} + m_{2,f} + \cdots + m_{n,f}) + \xi \cdot \left(\sqrt{\sigma_{1,f}^2 + \sigma_{2,f}^2 + \cdots + \sigma_{n,f}^2}\right). \tag{51}$$

Heat demand of 99% reliability level:

$$\dot{Q}_{m,f} = (m_{1,f} + m_{2,f} + \cdots + m_{n,f}) + 2.33 \cdot \left(\sqrt{\sigma_{1,f}^2 + \sigma_{2,f}^2 + \cdots + \sigma_{n,f}^2}\right). \tag{52}$$

Let us define the coincidence factor as the ratio of heat demand defined by formula (50), and to be obtained by the simple aggregation of individual design heat demand figures:

$$e = \frac{\dot{Q}_{m,f}}{\sum_{i=1}^{n} \dot{Q}_{m,i,f}} = \frac{\sum_{i=1}^{n} m_{i,f} + \xi \cdot \sqrt{\sum_{i=1}^{n} \sigma_{i,f}^2}}{\sum_{i=1}^{n} (m_{i,f} + \xi \cdot \sigma_{i,f})}. \tag{53}$$

Let consumers be identical and let us examine the coincidence factor if $n \to \infty$.

From expression (52), as $m_{1,f} = m_{2,f} = \cdots = m_{f,n} = m_f$

$$\lim_{n\to\infty} e = \frac{nm_f + \xi\sqrt{n}\sigma_f}{n\left(m_f + \xi\sigma_f\right)} = \frac{m_f}{m_f + \xi\sigma_f} + \frac{\xi\sigma_1}{\sqrt{n}\left(m_f + \xi\sigma_f\right)} = \frac{m_f}{m_f + \xi\sigma_f}. \tag{54}$$

Coincidence factor value trends in various cases were presented in Ref. [10]. Fig. 2 shows coincidence factor trends for a few selected values. Probability theory discussion puts the specification of simultaneous heat demand in an absolutely different perspective, making them exact.

5. Probability analysis of heat demand for heating based on measurements, by regression calculations

In the previous section, the uncertainty of heat demand for heating was stated synthetically, using the uncertainties of the factors in formula 1. In case of existing systems, statistics of real heat demand measured are available a function of external temperatures. In these cases, regression analysis should be applied for planning heat demand.

The linear regression correlation between the average daily heat demand ($\dot{Q}$) and average daily external temperature ($\bar{t}$) is:

$$\dot{Q} = A\bar{t} + B\,[MW]. \tag{55}$$

where$\dot{Q}$ stands for the daily average heat demand [MW],$\bar{t}$ for the average daily external temperature (°C),

$$A = \frac{n\sum_{i=1}^{n} \bar{t}_2 \dot{Q}_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} \dot{Q}_i}{n\sum_{i=1}^{n} \bar{t}_i^2 - \left(\sum_{i=1}^{n} \bar{t}_i\right)^2}\,[\mathrm{MW}/^\circ\mathrm{C}], \tag{56}$$

$$B = \frac{\sum_{i=1}^{n} \dot{Q}_i - A\sum_{i=1}^{n} \bar{t}_i}{n}\,[\mathrm{MW}]. \tag{57}$$

Obviously, the straight line defined specifies the expected value of daily average heat output associated with a given external temperature. However, it is also important to know what is the value that cannot be exceeded by the actual value at a given reliability level.

A closer scrutiny of this issue is provided by defining residual standard deviation:

$$S_{Q\bar{t}} = \sqrt{\frac{\sum_{i-1}^{n} \dot{Q}_i^2 - B\sum_{i-1}^{n} \dot{Q}_i - A\sum_{i-1}^{n} \bar{t}_i \dot{Q}_i}{\nu}}\,[\mathrm{MW}]. \tag{58}$$

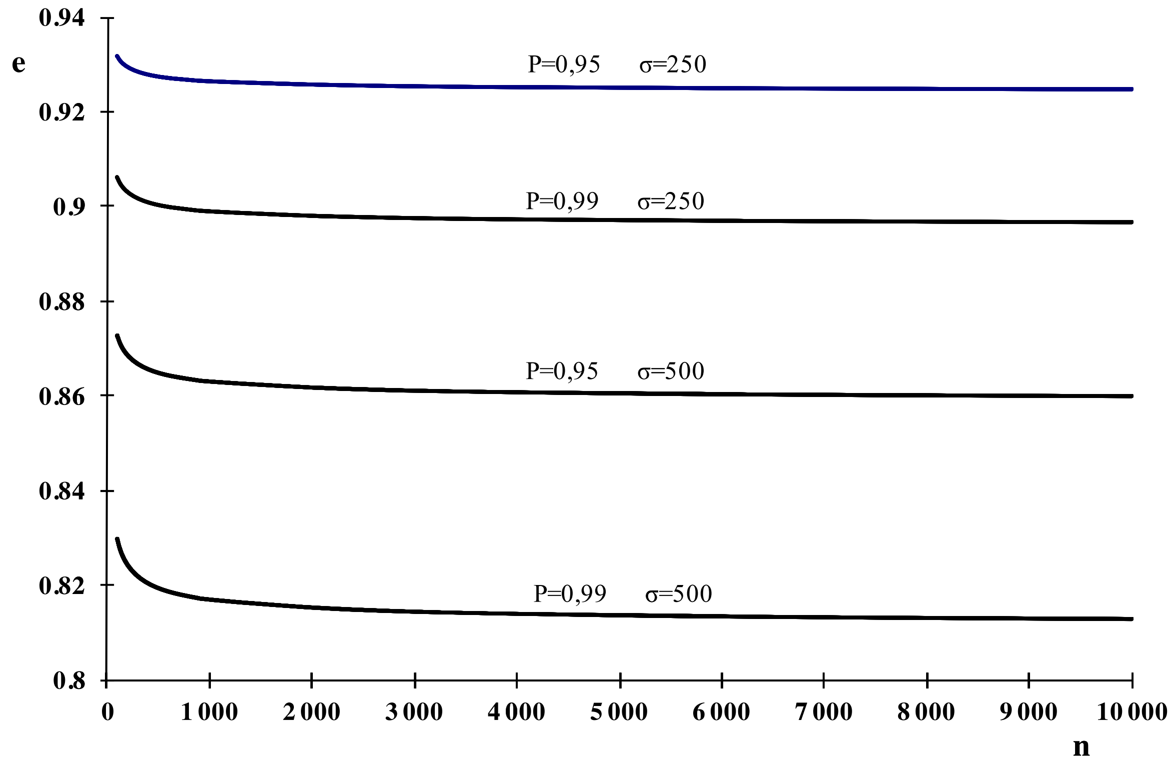

Fig. 2. Coincidence factor trends for a few selected values.

After defining standard deviation, the modulus (confidence interval in 95% significant level)

$$K = 1.96 \quad S_{\dot{Q}\bar{t}} \sqrt{\frac{1}{n} + \frac{\left(t_{k,0} - \frac{1}{n}\sum_{i=1}^{n}\bar{t}_i\right)^2}{\sum_{i=0}^{n}\left(\bar{t}_i - \frac{1}{n}\sum_{i=1}^{n}\bar{t}_i\right)^2}}\,[\mathrm{MW}] \tag{59}$$

can be calculated, which actually defines this interval.

So the nominal heat demand is:

$$\dot{Q}_o = \dot{Q}_n + K\,[MW]. \tag{60}$$

If $\bar{t}_b$ can be determined, then

$$\dot{Q}_n = \left(At_{k,0} + B\right)\frac{20 - t_{k,0}}{\bar{t}_b - t_{k,0}}\,[\mathrm{MW}]. \tag{61}$$

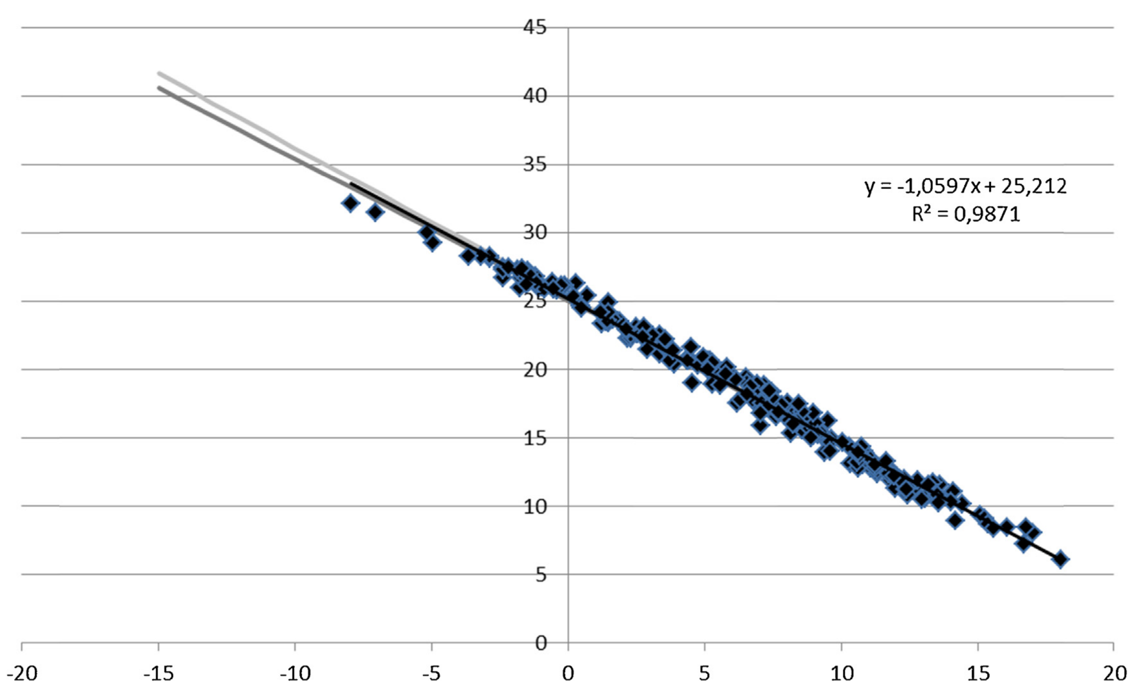

Fig. 3. Regression line for heat demand as a function of external temperature in the district heating system of the town of Kazincbarcika.

Table 4
Heat demand as a function of external temperature in the district heating system of the town of Kazincbarcika.

External temperature [°C]	Forward temperature [°C]	Backward temperature [°C]	Heat demand [MW]
−13	115.1	58.8	39.0
−12	113.7	58.6	37.9
−11	112.4	58.5	36.9
−10	111.0	58.3	35.8
−9	109.6	58.1	34.7
−8	108.3	58.0	33.7
−7	106.9	57.8	32.6
−6	105.5	57.6	31.6
−5	104.2	57.5	30.5
−4	102.8	57.3	29.5
−3	101.4	57.1	28.4
−2	100.0	56.9	27.3
−1	98.7	56.8	26.3
0	97.3	56.6	25.2
1	95.9	56.4	24.2
2	94.6	56.3	23.1
3	93.2	56.1	22.0
4	91.8	55.9	21.0
5	90.5	55.8	19.9
6	89.1	55.6	18.9
7	87.7	55.4	17.8
8	86.4	55.3	16.7
9	85.0	55.1	15.7
10	83.6	54.9	14.6
11	82.3	54.8	13.6
12	80.9	54.6	12.5

If it cannot, then

$$\dot{Q}_n = At_{k,0} + B\,[\text{MW}]. \quad (62)$$

The function defined makes it possible to derive heat demand in case of a discretionary external temperature by substituting t for $t_{k,o}$.

A statistical analysis of heat demand for heating was performed on each large district heating system in Hungary. A regression function according to formula (55) was adjusted to heat demand measured as a function of external temperature. By the extrapolation of the regression functions, the expected value of system heat demand was determined for the dimensioning status (−15 °C in Hungary). Heat demand can be determined by calculating the value of residual standard deviation for the required reliability level – 95% or 99% (the risk level being 5% or 1%, respectively).

6. Example

Let us demonstrate as an example, based on studies by Tibor Lakatos [10], the statistical analysis and regression lines of the heat supply system of a town in Hungary (Kazincbarcika). The nominal design heat demand was 65 MW. Operating practice proved that this was oversized. The regression line yielded an expected value of 39 MW for −13 °C. By calculating the residual value, the confidence interval was determined for the same external temperature, amounting to ±0.54 MW. This means that there is a 95% probability that actual heat demand will not exceed 39.54 MW at −13 °C external temperature. It should be noted that the likelihood of −13 °C as daily average temperature is ~1%. Our analyses disclosed that the system operates with very large reserves and can be extended by new consumers (Fig. 3 and Table 4).

7. Conclusions

Our study has pointed out that the methods for determining heat demand as set out by national standards are deterministic.

- Heat demand for central and district heating are probability variables containing measurable uncertainties.
- Two methods have been presented for the mathematical treatment of this issue.
- One of the methods is synthetic where the algebra of probability variables is applied.
- The other method is based on the regression analysis of measured consumption values.

The function specified indicates expected heat demand values, and deviations from expected values can be determined by residual standard deviation.

Depending on their size, district heating systems represent an asset value of 10–100 billion EUR in Hungary – and of course in European countries where they are widespread. The asset value of the heating system of large office buildings also may reach 5–10 million EUR.

The methodology for determining the heat demand for a dimensioning status to form a basis for design is outdated, unscientific, and reflects an unsustainable approach.

In formulas used for determining heat demand, designers do not analyze the uncertainties of meteorological and building physics characteristics and errors, and dimensioning standards do not contain any instructions and methodologies for this.

Standards reflect the so-called deterministic approach: risk-based dimensioning, a probability theory basis, the mathematically supported, exact error analysis of parameters and results, the confidence band, and reliability level surveys are outside the scope of design practice.

In other branches of mechanical engineering practice – design of structures, machine design, statics, transport design, measurement theory, measurement design, etc. – it is accepted, even required to replace the deterministic approach by risk-based design.

The methods presented in our study can be used for avoiding unreasonable oversizing or underdimensioning, losses caused whereby can reach 20–30% both in terms of investment development and operation according to European observations. The repeated valuation and qualification of existing systems can provide enormous opportunities for extending the pool of district heating consumers without capacity expansion, purely by exploring latent reserves.

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