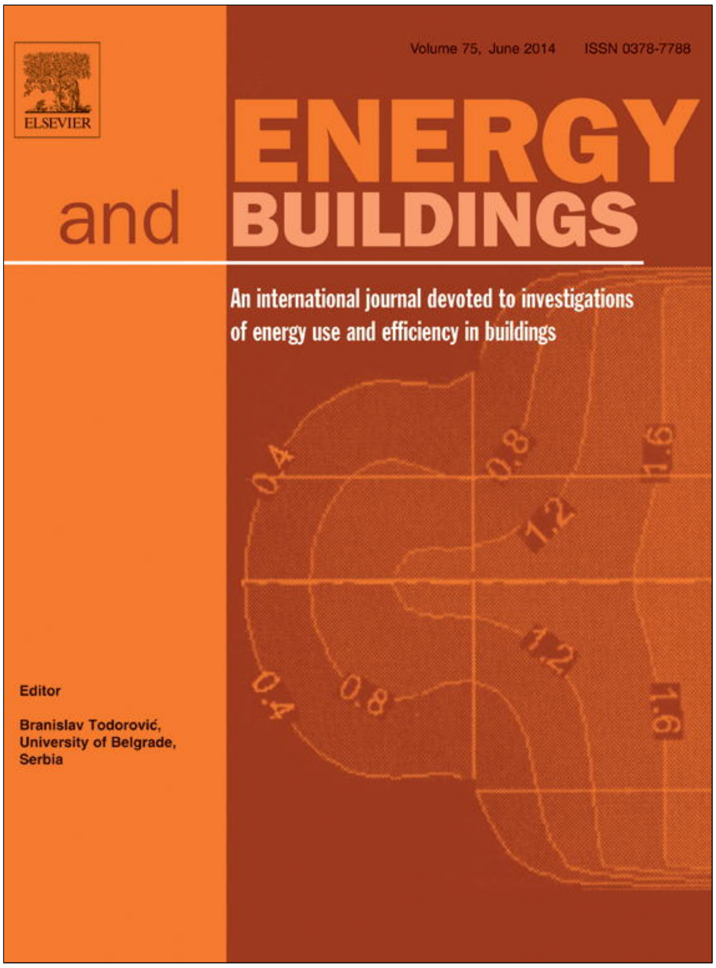
Volume 75, June 2014 ISSN 0378-7788
ELSEVIER
ENERGY
and BUILDINGS
An international journal devoted to investigations of energy use and efficiency in buildings
Editor
Branislav Todorović,
University of Belgrade,
Serbia

Energy and Buildings 75 (2014) 483–492

Contents lists available at ScienceDirect

Energy and Buildings

journal homepage: www.elsevier.com/locate/enbuild

Probability theory description of domestic hot water and heating demands

László Garbai[a], Andor Jasper[a], Zoltán Magyar[b,*]

[a] Department of Building Service and Process Engineering, Budapest University of Technology and Economics, Hungary
[b] Department of Building Energetics and Building Service Engineering, Budapest University of Technology and Economics, Hungary

ARTICLE INFO

Article history:
Received 16 January 2014
Accepted 28 January 2014

Keywords:
DHW
Probability theory
Heat demands
Duration curve
External temperature
Degree hours
Normal distribution
Confidence interval

ABSTRACT

The paper presents the methodology for the risk-based dimensioning of domestic hot water (DHW) and heating demands. It is demonstrated that they are stochastic variables. DHW demands are specified on the basis of hot water consumption statistics in Hungary. The standard values of heat demands for heating are based on the duration curve of external air temperatures. This study discusses daily mean temperature distribution in the heating season – between 15 October and 15 April in Hungary – and the description of its duration curve sorted by time. The authors verify the assumption that the duration curve can be described by normal distribution mathematically with high precision. Thereby they somewhat put an end to the process of mathematical experimentation on what function should be used to describe the frequency and duration curve of external daily mean temperatures in the heating season.

1. Introduction

Most building engineering systems satisfy heating and air conditioning as well as cold and hot water demands. Heating demands are meteorological (weather-based) factors, primarily dependent on external temperature. DHW demands depend on the lifestyle and hygiene levels of consumers. They show a time-of-day course and change randomly. The operation of building engineering systems requires the satisfaction of stochastically changing demands (changing randomly in the course of time). And system design is targeted to install system equipment with dimensions and capacities to enable them to satisfy maximum demand levels occurring rarely and at low rates of probability at the safety levels required. Risk-based dimensioning is applied in building engineering system installation design. Chances of both over dimensioning and under dimensioning must be avoided. In the course of risk-based dimensioning, weather factors can be considered as stochastic variables and their frequency of occurrence and probability are explored. The occurrence of weather factors at a very low probability rate is considered as a so-called nominal state of dimensioning, which is a load state of 1% risk for the most part.

The risk-based dimensioning methodology is obviously a well-known discipline; its proposed application in building engineering was presented in an earlier publication [1]. Both operation and installation design are optimization tasks controlled by economic target functions nowadays. Systems must be designed in a way that the aggregate cost of system development and operation should reach a minimum and should enable the satisfaction of demands at a minimum cost consciously and verifiably. In this respect, our study presents a methodology for the risk-based determination of DHW demands and heating demands. It is demonstrated that both hot water demands and heating demands are stochastic variables and their distribution is described by standard distribution.

2. The probability feature of domestic hot water demands

Previously literatures do not investigate the DHW and heating demands based on probability theory [6–12].

DIN 4708 standard describes the calculation method of the DHW demand based on deterministic point of view. In the standard is no risk-based approach. This paper presents the necessity of aspect change and gives solution [5].

In respect of consumer groups of a discretionary number of apartments, the quantity and intensity of consumption at any time of day and for any period present random and unpredictable fluctuations, therefore they can be considered as stochastic variables and can be described by probability function relations. In case of

* Corresponding author. Tel.: +36 309 642 500.
E-mail addresses: zmagyar@invitel.hu, magyar@egt.bme.hu (Z. Magyar).

http://dx.doi.org/10.1016/j.enbuild.2014.01.050
0378-7788/

Nomenclature

Q_{DHW}	DHW consumption
n	the number of apartments
m_{DHW}	the expected value of DHW consumption
σ_{DHW}	the standard deviation of DHW consumption
ξ	the abscissa value of standard normal distribution at the required reliability level (e.g. 99%)
$\dot{V}_{average}$	the daily average consumption intensity
$\dot{V}_{DHWEC}$	the capacity of the DHW heat exchanger
τ_{disch}	duration of discharge
λ_{St}	Student distribution
s^*	adjusted empirical standard deviation
m	the expected value of daily mean external temperature
σ	the dispersion of daily mean external temperature

consumption for any period of time, consumption is characterized by its density and distribution functions. The distribution function is generally normal, determined by two of its parameters, namely the expected value and standard deviation, which are to be specified by measurements. In case of domestic hot water consumption and intensity for any period of time, we are interested in the so-called standard values. Standard values are characterized by the fact that by definition, it can be expected at a 95 or 99% reliability rate that no higher values occur in the consumption period examined. The standard value of consumption for a certain period of time at a time of day:

$$Q_{DHW} = m_{DHW} + \xi\sigma_{DHW}. \tag{1}$$

Remark. Based on measurements and statistic tests DHW consumption is described by normal distribution of probability variables. We introduced the standardized form of the probability variable as:

$$\xi = \frac{Q - m_{DHW}}{\sigma_{DHW}}.$$

Values of standard normal probability distribution are in the table [13]. ξ value could be calculation from the table [13]. The design DHW value could be calculation in given reliability level.

Standard consumption for a period of time, using the simplifying concept of the so-called apartment unit (average apartment), in function of the number of apartments, at a time of day:

$$Q_{DHW}(n) = n\, m_{DHW} + \xi\sqrt{n}\sigma_{DHW}, \tag{2}$$

where Q is DHW consumption, n is the number of apartments, m_{DHW} is the expected value of DHW consumption, σ_{DHW} is the standard deviation of DHW consumption, ξ is the abscissa value of standard normal distribution at the required reliability level (if e.g. 99%, $\xi = 2.33$).

The expected value and standard deviation of DHW consumption of a required period can be determined by means of mathematical statistics based on measurements. It is expedient to carry out measurements for various types of buildings and a variety of numbers of apartments, and to correlate the figures of expected value and standard deviation to an apartment unit.

Fig. 1 shows the qualitative image of standard consumptions in function of the number of apartments.

The methodology for determining the standard values of domestic hot water consumption on a probability theory basis was developed by Garbai and Lakatos [2].

Intensity curves and calculation formulas of standard domestic hot water consumptions of different durations are summarized in the German (DIN 4708) and the Hungarian standards. The Hungarian standard was developed in the 1980s on the basis of unmeasured hot water consumption statistics. It is not recommended any longer to apply the values and formulas published therein as they yield considerably higher consumption figures than actual figures based on currently measured consumption rates.

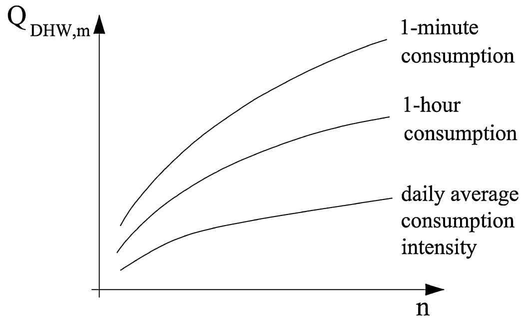

Fig. 1. Qualitative of standard domestic hot water consumption.

In 2004 and 2005, the hot water consumption of a number of buildings was measured by Budapest District Heating Company on an on-going basis, the figures yielded were processed and values and calculation formulas of 99% reliability were worked out in order to determine the duration curves of consumptions and consumption intensities of various periods of time (minute-based consumptions) [4].

The following expression can be used to describe the 99% reliability level sorted duration curve function of consumption intensities.

$$\dot{V} = A\,\tau^{B} + c\,G\,\tau \quad [\mathrm{l/\,min}], \tag{3}$$

where

$$A = 28.623\,\dot{V}_{average}^{0.4893}, \tag{4}$$

$$B = -0.27\,\dot{V}_{mean}^{-0.224} + 0.000813\,\dot{V}_{average}, \tag{5}$$

$$C = -0.00265\,\dot{V}_{average} - 0.0135. \tag{6}$$

$\dot{V}_{average}$ is the daily average consumption intensity [l/min].

The formula expresses the duration of a consumption intensity value or a higher value at 99% reliability level.

The daily average consumption intensity of 99% reliability level can be determined by the following formula:

$$\dot{V}_{average} = n\cdot a + 2.33\sigma\sqrt{n}, \quad \dot{V}_{average} = 0.135\,n + 0.3\sqrt{n} \quad [\mathrm{k/min}]. \tag{7}$$

The formula indicates that the probability distribution of daily average consumption intensities follows normal distribution. The expected value for 1 apartment is $a = 0.135$ l/min, and the standard deviation for 1 apartment is $\sigma = 0.128$ l/min.

Based on our investigations, it has been demonstrated that the quantity of hot water consumed in a peak consumption period of discretionary duration also indicates normal distribution. Table 1 shows the expected value and standard deviation of the quantity of hot water consumed in a given period in function of the peak consumption period. Fig. 2. shows consumption.

Units of measure applied in the table:

$$a\left[\frac{1}{\mathrm{apartment}}\right],$$

$$\sigma\left[\frac{1}{\mathrm{apartment}}\right].$$

Quantity of consumption at 99% reliability level in case of an n number of apartments, using the data in Table 1: $V = n\cdot a + 2.33\sigma\sqrt{n}$.

Table 1
Expected value and standard deviation of the quantity of hot water consumed in various peak consumption periods.

Consumption period (min)	Expected value of consumption quantity a	Standard deviation of consumption quantity σ	Consumption period (min)	Expected value of consumption quantity a	Standard deviation of consumption quantity σ	Consumption period (min)	Expected value of consumption quantity a	Standard deviation of consumption quantity σ
1	−0.220	6.572	41	3.657	123.777	81	9678	207.247
2	−0.281	11.467	42	3.796	126.085	82	9836	209.150
3	−0.298	15.859	43	3.937	128.378	83	9995	211.045
4	−0.290	19.947	44	4.078	130.657	84	10.154	212.933
5	−0.263	23.821	45	4.220	132.922	85	10.314	214.813
6	−0.223	27.530	46	4.362	135.174	86	10.473	216.687
7	−0.173	31.107	47	4.506	137.412	87	10.633	218.554
8	−0.114	34.572	48	4.650	139.638	88	10.793	220.414
9	−0.047	37.943	49	4.794	141.851	89	10.953	222.267
10	0.026	41.232	50	4.939	144.051	90	11.114	224.113
11	0.105	44.447	51	5.085	146.240	91	11.275	225.953
12	0.188	47.597	52	5.232	148.417	92	11.436	227.786
13	0.276	50.687	53	5.379	150.582	93	11.597	229.613
14	0.368	53.724	54	5.526	152.736	94	11.758	231.433
15	0.463	56.710	55	5.674	154.879	95	11.920	233.247
16	0.562	59.651	56	5.823	157.011	96	12.082	235.054
17	0.664	62.550	57	5.972	159.132	97	12.244	236.855
18	0.768	65.408	58	6.122	161.243	98	12.406	238.650
19	0.875	68.230	59	6.272	163.343	99	12.568	240.439
20	0.985	71.016	60	6.423	165.433	100	12.731	242.222
21	1.097	73.770	61	6.574	167.514	101	12.894	243.999
22	1.211	76.492	62	6.726	169.584	102	13.057	245.769
23	1.326	79.184	63	6.878	171.645	103	13.220	247.534
24	1.444	81.849	64	7.030	173.696	104	13.383	249.293
25	1.564	84.486	65	7.183	175.738	105	13.547	251.046
26	1.685	87.098	66	7.336	177.771	106	13.710	252.794
27	1.808	89.685	67	7.490	179.795	107	13.874	254.535
28	1.932	92.248	68	7.644	181.810	108	14.038	256.271
29	2.058	94.789	69	7.798	183.816	109	14.202	258.002
30	2.185	97.308	70	7.953	185.813	110	14.366	259.727
31	2.313	99.806	71	8.108	187.802	111	14.530	261.446
32	2.443	102.284	72	8.264	189.782	112	14.695	263.160
33	2.573	104.743	73	8.420	191.755	113	14.860	264.868
34	2.705	107.182	74	8.576	193.719	114	15.024	266.571
35	2.838	109.604	75	8.732	195.675	115	15.189	268.269
36	2.972	112.007	76	8.889	197.623	116	15.354	269.961
37	3.107	114.394	77	9.046	199.563	117	15.519	271.648
38	3.243	116.763	78	9.204	201.495	118	15.685	273.330
39	3.380	119.117	79	9.361	203.420	119	15.850	275.007
40	3.518	121.455	80	9.519	205.337	120	16.016	276.678

The formulas presented can be used to determine, for an $n > 20$ number of apartments, the heat exchanger and DHW storage tank required for meeting DHW demands at a 99% reliability level. It is obviously uneconomical to install a DHW heat exchanger required to satisfy very high DHW minute peaks. The output to satisfy DHW minute peaks could not even be satisfied by the hot water capacity of the system. Therefore DHW peaks of short periods can be satisfied at every caloric centre by the coordinated operation of the DHW heat exchanger and the DHW storage tank. For a given number of apartments, the DHW heat exchanger and the DHW storage tank are selected as follows.

Formula (7) is used to determine the daily average HMW output demand ($\dot{V}_{average}$). This is used to calculate constants A, B and C in formula (3).

Then a period is specified during which the DHW storage tank is operated, during which the DHW storage tank is discharged. During

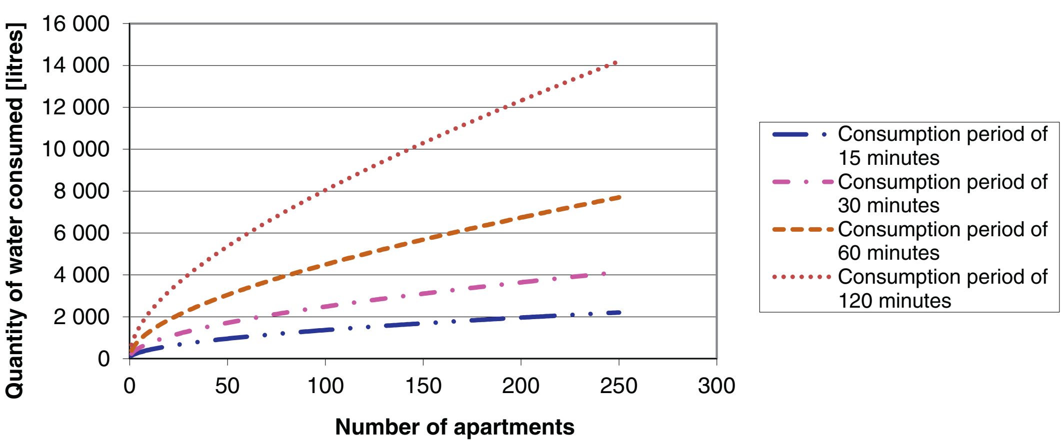

Fig. 2. Consumption for various periods in function of the number of apartments.

Table 2
Statistical features of daily average temperatures of various years for Budapest.

Year	1997/1998	1998/1999	1999/2000	2000/2001	2001/2002	2002/2003	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
Annual average [°C]	4.822	3.157	3.623	5.583	3.687	2.648	3.480	3.789	3.187	7.362	4.727	5.313	4.200
Dispersion [°C]	4.757	5.943	4.993	4.960	6.080	6.411	5.231	6.038	5.392	4.335	4.767	5.906	5.589
Average of annual average values [°C]			4.28										
Dispersion [°C]			1.23										

this period, the DHW storage tank and the heat exchanger work together to satisfy DHW demands. During this period, the DHW heat exchanger runs at full capacity. The capacity of the DHW heat exchanger can be determined by formula (3). Volume: $V_{HE} = \dot{V}$.

In order to satisfy a demand larger than that, the storage tank must be discharged. The heat stored in the DHW storage tank can be produced by the integration of the expression:

$$Q = \left(\int_0^{\tau_{disch}} \dot{V} d\tau - \dot{V}_{HE} \tau_{disch} \right) \cdot \rho \cdot c \cdot t_{DHW} \tag{8}$$

In detail:

$$Q = \left(\int_0^{\tau_{disch}} (A\tau^B + c\tau) d\tau - \dot{V}_{HE} \tau_{disch} \right) \cdot \rho \cdot c \cdot t_{DHW}, \tag{9}$$

$$Q = \left(\frac{A}{B+1} \tau_{disch}^{B+1} + \frac{C}{2} \tau_{disch}^2 - \dot{V}_{HE} \tau_{disch} \right) \cdot \rho \cdot c \cdot t_{DHW}. \tag{10}$$

Correlation between the heat stored and the volume of the DHW storage tank:

$$Q = V \rho c t_{DHW}.$$

From this, the required volume of the DHW storage tank is:

$$V = \frac{Q}{\rho c t_{DHW}}.$$

The required size of the DHW storage tank and the output of the DHW heat exchanger should be selected by economic optimum calculations.

The diagrams enclosed (Figs. 3–5) show the characteristic course of daily hot water consumption, the daily duration curves of sorted consumption intensities, as well as dimensioning diagrams to determine the required heat exchanger capacity in function of the number of apartments and in function of the size of the selected storage tank as a parameter.

3. Probability distribution of external daily mean temperatures

A very important, mostly graphically displayed tool of building engineering design and operation is the so-called duration curve to indicate the frequency of occurrence and continuity of external temperatures. Duration curves have been applied in design and operation for several decades. Their practical significance is given by the fact that they can be used to specify the very low and short-term external temperature range for which the capacity of heat generation and transfer equipment is designed, while the area below the curve is proportionate to the heat generated/consumed. Although its practical significance is beyond doubt, the duration curve has never been described theoretically by exact mathematical means, and no experiment of probability theory modelling has been conducted. (Various authors applied a deterministic polynomial description. There have been no experiments on the frequency of occurrence and risk of a duration curve described by a function.) Fig. 6 shows the duration curves of daily average temperatures for the heating periods in the years 1997–2011, based on the meteorological statistics of Budapest. Visually, the duration curves display normal probability distribution. In respect of the likelihood of the absence of external temperatures higher than the temperature examined, an approximative value is yielded by the quotient of the continuity period associated with such temperature and the entire period of heating.

Let us consider the external daily mean temperature as a stochastic variable.

Let us generate, according to Fig. 6, the duration curve from the average of daily mean temperatures associated with the periods of heating in 1997–2011.

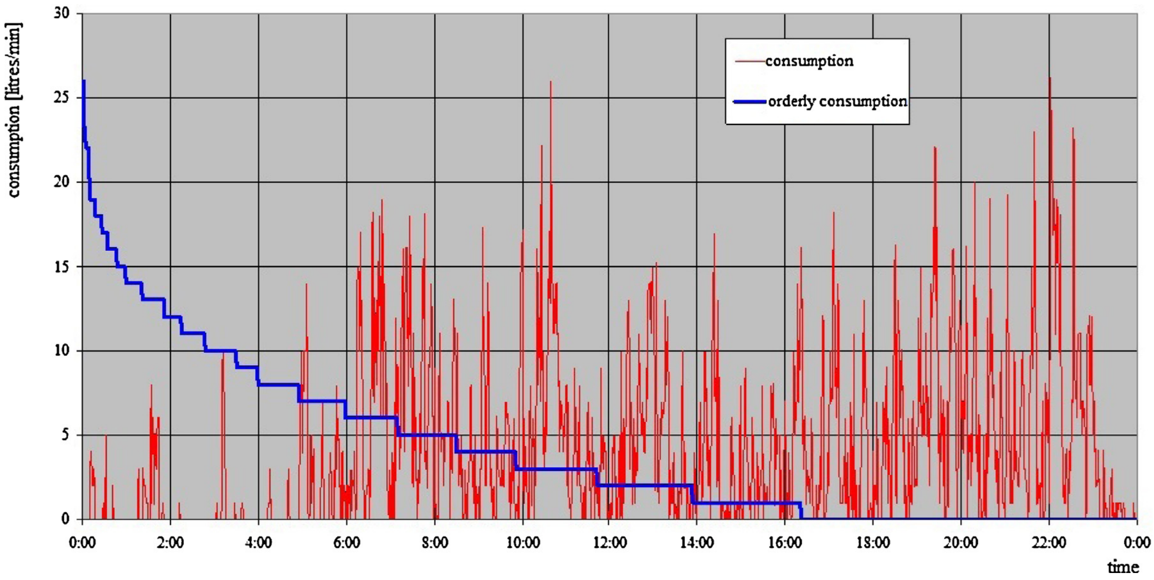

Fig. 3. Daily course and sorted duration curve of DHW consumption.

Let us generate the empirical probability distribution of daily mean temperatures from the duration curve. This is shown in Figs. 7 and 8. It shows the normal distribution the expected value and standard deviation of which have been calculated from the values of the average duration curve. Momentums of this distribution: $m = 4.28\,°C$, $\sigma = 5.44\,°C$ (Table 2).

Probability distribution has been standardized, showing an almost perfect correspondence with standardized normal distribution (see Fig. 7). A Ryan-Joiner test has been conducted to verify that the probability distribution of daily average temperatures really indicates standard deviation. As a result of the test, the correlation coefficient is $r^* = 0.997$. As it is considerably higher than the $r^* = 0.9600$ criterion value, our assumption is verified [3].

Probability distribution function of daily average temperatures for Budapest:

$$P(t_k \le \eta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^{\eta} e^{-(x-m)^2/2\sigma^2} dx = \frac{1}{5.44 \cdot \sqrt{2\pi}} \cdot \int_{-\infty}^{\eta} e^{-(x-4.28)^2/2\times 5.44^2} dx \quad (11)$$

Function values can be calculated from the tabular values of standardized normal distribution. The value

$$\xi = \frac{\eta - m}{\sigma} = \frac{\eta - 4.28}{5.44} \quad (12)$$

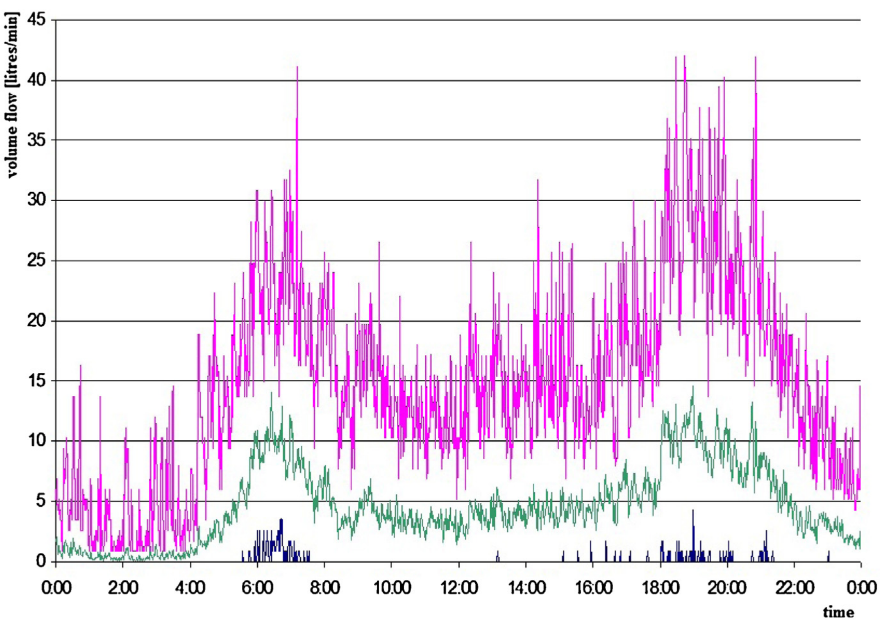

Fig. 4. Minimum, average and maximum values of DHW consumption for 28 days, Budapest in a typical building.

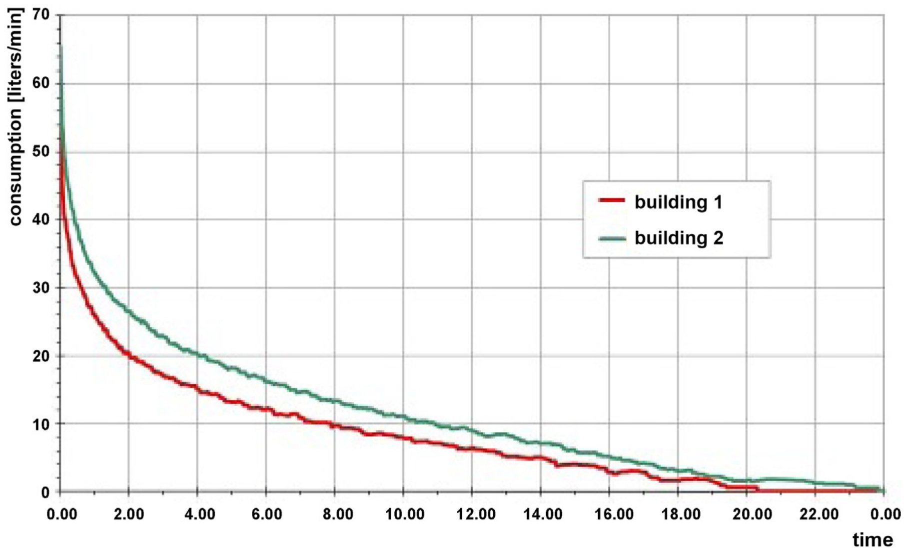

Fig. 5. 99% reliability level duration curve of two buildings of 65 apartments.

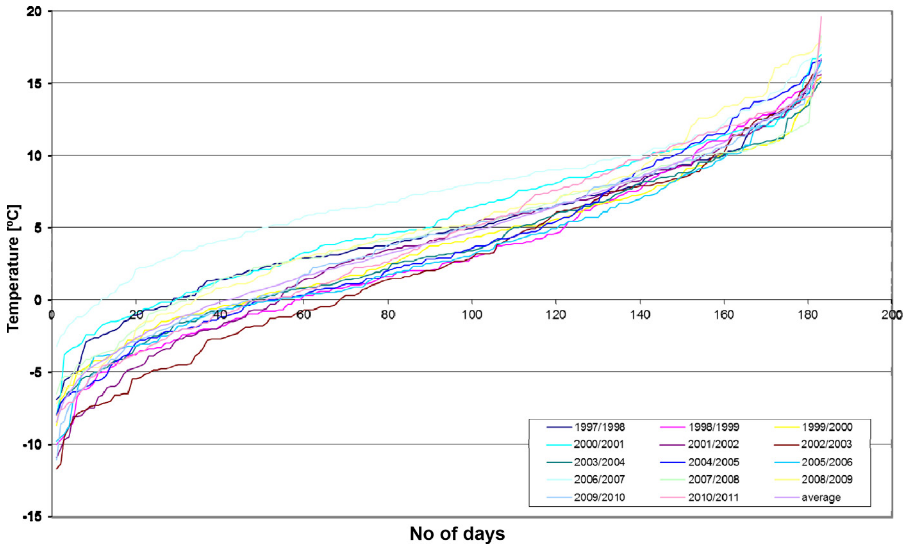

Fig. 6. Duration curve of daily average external temperatures in Budapest for periods of heating in 1997–2010.

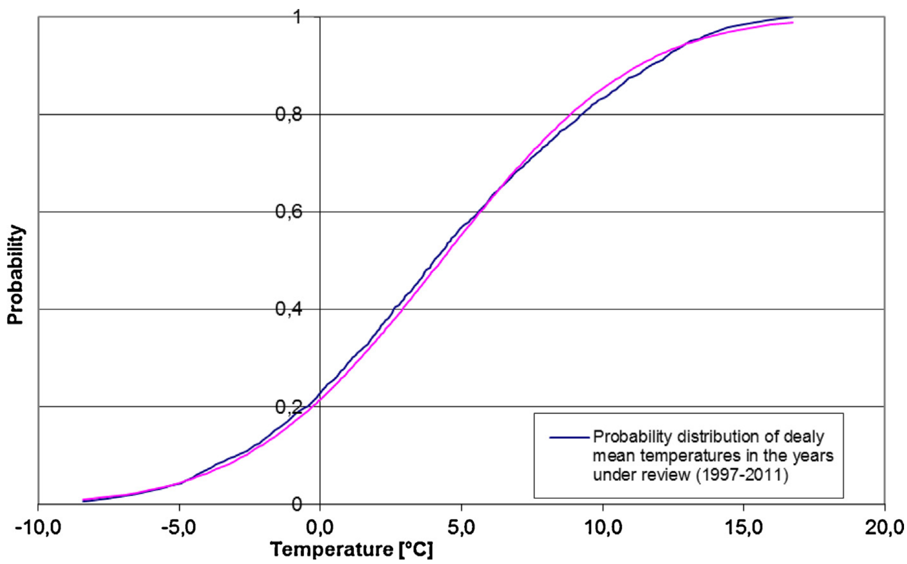

Fig. 7. Comparison of the probability distribution of daily average temperatures measured in Budapest to normal distribution.

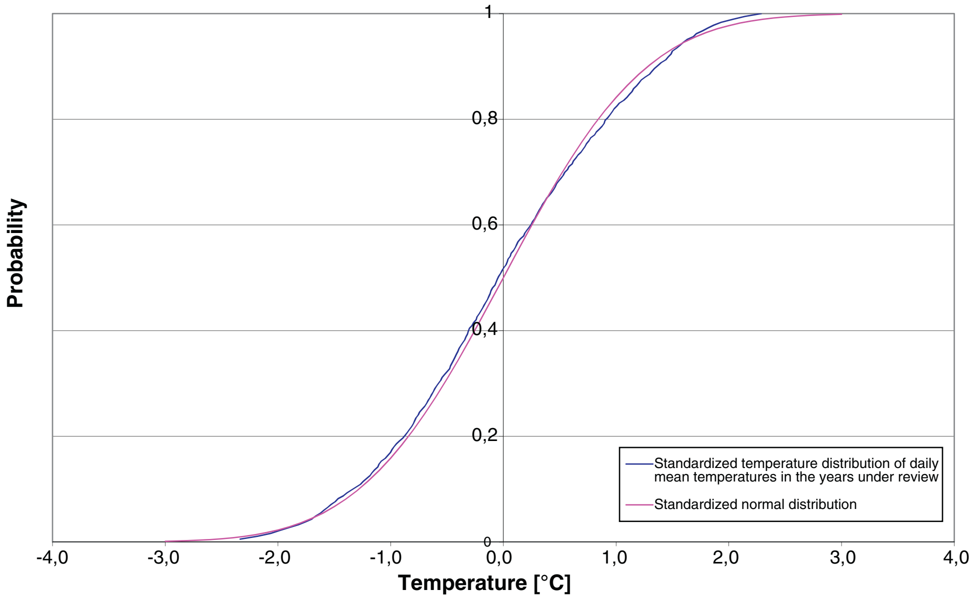

Fig. 8. Comparison of the standardized probability distribution of daily average temperatures to standardized normal distribution.

is produced and it is checked in the table for standardized normal distribution what is the probability distribution value associated with the value of η which yields the value $P(t_k \le \xi)$ sought for.

Standardized normal distribution function:

$$\phi(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi} e^{-t^2/2} dt \tag{13}$$

Standardized normal distribution figures can be found in [2], for instance. Table 3 shows the duration curve values of daily mean temperatures calculated as above.

4. Statistical testing for distribution momentum errors

Obviously, it can be assumed with good reason that daily mean temperature values follow normal distribution for any geographical area in respect of the heating period, as well as that they have different expected values and standard deviation figures which can be determined from the meteorological statistics of the geographical area (country, region, settlement) examined.

The values thus specified contain errors. Both the expected value and the standard deviation of external daily mean temperatures are stochastic variables, to be estimated by the figures of empirical expected value and standard deviation. Information on the connections between the empirical expected value and standard deviation and the respective theoretical values is provided by the confidence interval. The magnitude of errors can be specified from empirical values by statistical testing at a pre-determined reliability level – 95%, . . ., 99%. The confidence interval is the range of values which includes theoretical values at the required reliability level.

4.1. Determining errors in expected values

The standard deviation of expected values as stochastic variables is unknown; it is estimated on the basis of the sample by undistorted estimation, that is, empirical standard deviation s^*. Now the variable is not of normal distribution any longer: it is characterized by Student distribution [13]. The Student distribution table can be used for determining the λ_{st} value to delimit the confidence interval. This is expressed by the following formula:

$$\left(m - \lambda_{st} \frac{s^*}{\sqrt{n}};\ m + \lambda_{st} \frac{s^*}{\sqrt{n}} \right) \tag{14}$$

where n is the number of measurement points (number of days in heating season).

4.2. Determining standard deviation errors

A confidence interval for the variance of a sample of n elements can be constructed by taking the distribution of the probability variable $n(S^2/\sigma^2)$ as a starting point. It is known that the variable $n(S^2/\sigma^2)$ follows a distribution χ^2 of $(n-1)$ degrees of freedom, which can be used for determining constants a_1, a_2 complying with the following criteria: $P(n(S^2/\sigma^2) < a_1) = \varepsilon/2$ and $P(n(S^2/\sigma^2) < a_2) = 1 - \varepsilon/2$ [13]. The χ^2 distribution is not symmetrical, therefore limits to satisfy $P(a_1 < n(S^2/\sigma^2) < a_2) = 1 - \varepsilon$ should be selected. Hence the confidence interval for σ^2, of reliability $1-\varepsilon$, is $n(S^2/a_2) < \sigma^2 < n(S^2/a_1)$. A confidence interval can also be yielded for parameter σ of the multitude; in this case, however, the ξ distribution table should be used.

5. Probability theory description of daily average degree hours

The probability distribution and duration curves of degree hours, that is, differences between daily mean temperatures and indoor heating temperatures play a very important part in building engineering design practice.

It is a dominant feature in the practical calculation of heat consumption for heating to determine the area below the duration curve of daily average temperatures. After demonstrating that the duration curve of daily average temperatures can be brought into an exact mathematical connection with the formulas of normal and standardized normal distribution, in consideration thereof the area below the duration curve can be calculated by the function to describe the probability distribution of daily mean temperatures.

Table 3
Duration curve values of daily mean temperatures for Budapest – expected value: $m = 4.28\,°C$; standard deviation: $\sigma = 5.44\,°C$.

Number of days	Daily mean temperature [°C]	Number of days	Daily mean temperature [°C]	Number of days	Daily mean temperature [°C]	Number of days	Daily mean temperature [°C]	Number of days	Daily mean temperature [°C]
1	−8.42	38	−0.35	75	2.77	112	5.79	149	9.54
2	−7.59	39	−0.25	76	2.84	113	5.87	150	9.62
3	−6.94	40	−0.14	77	2.96	114	5.94	151	9.76
4	−6.52	41	−0.06	78	3.01	115	6.01	152	9.87
5	−6.06	42	0.00	79	3.11	116	6.09	153	10.09
6	−5.60	43	0.06	80	3.19	117	6.18	154	10.25
7	−5.27	44	0.17	81	3.27	118	6.28	155	10.36
8	−4.95	45	0.24	82	3.35	119	6.36	156	10.49
9	−4.79	46	0.36	83	3.43	120	6.49	157	10.56
10	−4.54	47	0.49	84	3.51	121	6.60	158	10.71
11	−4.38	48	0.58	85	3.57	122	6.71	159	10.82
12	−4.20	49	0.65	86	3.64	123	6.79	160	10.94
13	−4.04	50	0.74	87	3.68	124	6.84	161	11.12
14	−3.84	51	0.82	88	3.76	125	6.95	162	11.34
15	−3.69	52	0.89	89	3.88	126	7.06	163	11.44
16	−3.44	53	0.96	90	3.94	127	7.16	164	11.56
17	−3.26	54	1.06	91	3.99	128	7.24	165	11.74
18	−3.06	55	1.15	92	4.06	129	7.38	166	11.89
19	−2.84	56	1.26	93	4.14	130	7.44	167	12.12
20	−2.62	57	1.32	94	4.21	131	7.61	168	12.24
21	−2.48	58	1.44	95	4.29	132	7.66	169	12.37
22	−2.32	59	1.56	96	4.41	133	7.79	170	12.44
23	−2.16	60	1.69	97	4.46	134	7.88	171	12.64
24	−2.06	61	1.73	98	4.51	135	8.03	172	12.81
25	−1.94	62	1.80	99	4.61	136	8.12	173	12.96
26	−1.78	63	1.87	100	4.66	137	8.22	174	13.13
27	−1.71	64	1.95	101	4.78	138	8.30	175	13.44
28	−1.61	65	2.01	102	4.86	139	8.41	176	13.59
29	−1.42	66	2.09	103	4.92	140	8.49	177	13.89
30	−1.29	67	2.14	104	4.99	141	8.68	178	14.14
31	−1.16	68	2.22	105	5.10	142	8.76	179	14.43
32	−1.09	69	2.32	106	5.26	143	8.91	180	14.79
33	−1.00	70	2.41	107	5.32	144	9.05	181	15.35
34	−0.89	71	2.47	108	5.41	145	9.14	182	15.93
35	−0.74	72	2.54	109	5.52	146	9.19	183	16.72
36	−0.58	73	2.59	110	5.61	147	9.32		
37	−0.46	74	2.64	111	5.69	148	9.43		

The duration curve to describe the temporal continuity of daily mean temperatures is a function of the probability distribution of daily mean temperatures.

$$\tau(t_k) = P(t_k \le \xi)\tau_f = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t_k} e^{-(x-m)^2/2\sigma^2} dx \cdot \tau_f \tag{15}$$

where τ_f is the entire period of heating. On average, this is 183 days in Budapest. $\tau(t_k)$ is the period while temperatures lower than the value of t_k occur during the winter heating period.

Fig. 9 shows the duration curves of daily mean temperatures and degree hours by different methods of display.

In building engineering practice, the heat required for heating in a certain period is shown by the area below the duration curve of degree hours, which is also a stochastic variable.

The area below the duration curve of daily average degree hours can be calculated as follows:

$$T_{T1} = 183 \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t_k} \int_{-\infty}^{t} e^{-(x-m)^2/2\sigma^2} dx\ dt \tag{16}$$

$$T_{T2} = 183 \cdot (20 - t_h) \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t_k} e^{-(x-m)^2/2\sigma^2} dx \tag{17}$$

In case of Budapest, if 12 °C is considered to be the boundary temperature for heating, and the average of daily average temperatures measured in 11 years is taken as $m = 4.277\,°C$ and their standard deviation as $\sigma = 5.439\,°C$, then the values of each area will be:

$$T_{T1} = 183 \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t_k} \int_{-\infty}^{t} e^{-(x-m)^2/2\sigma^2} dx$$

$$dt = 183 \cdot \frac{1}{5.439\sqrt{2\pi}} \int_{-\infty}^{12} \int_{-\infty}^{t} e^{-(x-4.277)^2/2\times 5.439^2} dx$$

$$dt = 7.9138 \cdot 183 = 1448.23 \quad \text{degreedays}$$

$$T_{T2} = 183 \cdot (20 - t_h) \cdot \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t_k} e^{-(x-m)^2/2\sigma^2}$$

$$dx = 183 \cdot (20 - 12) \frac{1}{5.439\sqrt{2\pi}} \int_{-\infty}^{12} e^{-(x-4.277)^2/2\times 5.439^2}$$

$$dx = 0.9224 \cdot 183 \cdot (20 - 12) = 1350.42 \quad \text{degreedays}$$

$$T_1 + T_2 = 1448.23 + 1350.42 = 2798.65 \quad \text{degreedays}$$

Integral values can be determined by a Maple or Excel software. In Excel, the following correlation should be entered:

$$\int_{-\infty}^{A} \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} e^{-(x-m)^2/2\sigma^2} dx\, dt =$$

"−2.138500000 + .5000000000*A + .5000000000*erf (.1300067625*A − .5560389231)*A − 2.138500000*erf

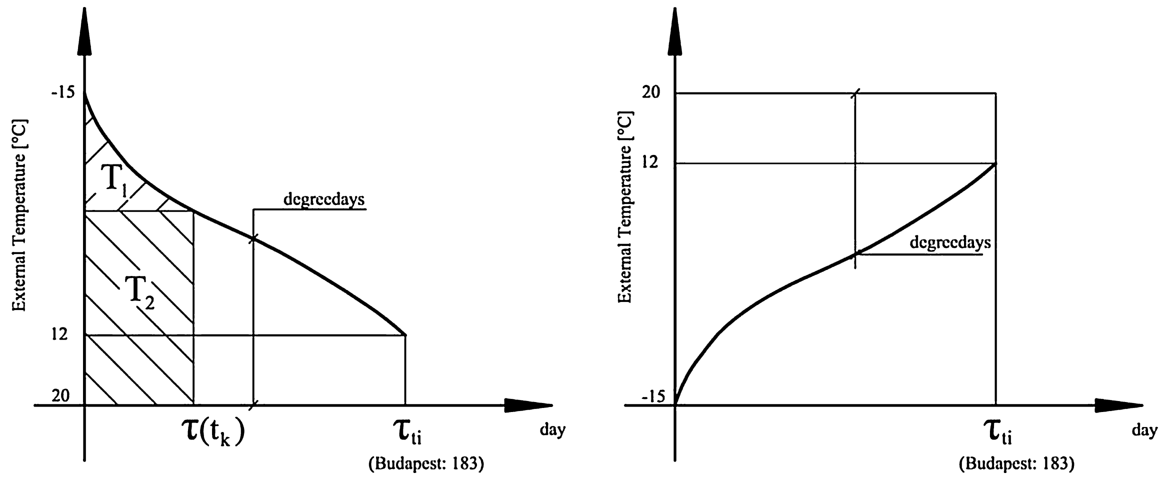

Fig. 9. Methods to display the duration curve of daily mean temperature.

(.1300067625*A – .5560389231) + 2.169847063*exp
(–.9000000000e – 19*(433355875.*A – 1853463077.)$\hat{2}$)"

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{A} e^{-(x-m)^2/2\sigma^2}dx = \text{“}.5001267876$$

$$+ .5001267876 * \mathrm{erf}(.1300067625 * A - .5560389231)\text{”}$$

As the Maple software applies an American notation system, the points should be exchanged to commas when using Excel in Hungarian, and the value of A should be modified to the value of the required boundary temperature for heating. In some Excel programmes, the Erf function is not included as a default function. In this case, the Analysis ToolPak checkbox should be marked in the Tools/Add-on Manager menu item.

6. Confidence interval of the expected value of external daily mean temperatures for Budapest

Statistics for 1900.2011 were taken as a basis for this calculation. The confidence interval of the expected value of daily mean temperatures was produced using empirical standard deviation and a value λ_{st} determined from the Student distribution table at a given reliability level. The empirical expected value is $m = 4.1$ °C, the radius of the confidence interval is $a = \lambda_{st}s^*/\sqrt{n} = 1.012$ °C, where $s^* = 5.453$, $n = 111$, $\lambda_{st} = 2.576$.

In respect of the confidence interval of the expected value,

- the lower boundary is 4.1 – 1.012 = 3.088
- the upper boundary is 4.1 + 1.012 = 5.112

Fig. 10shows the distribution functions calculated by applying the lower and upper boundaries of the expected value.

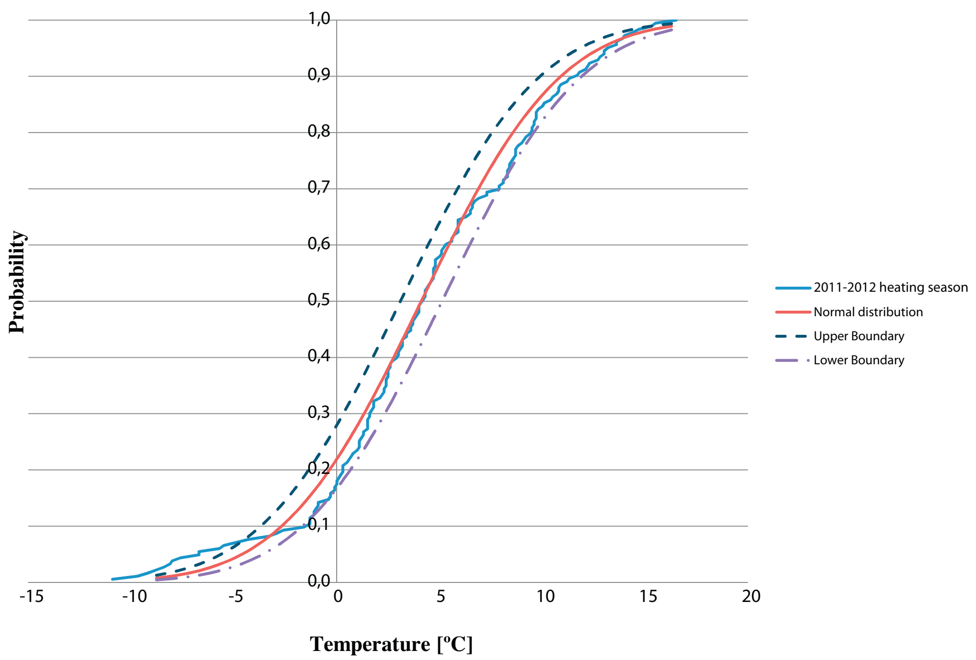

Fig. 10. 2011–2012 heating season within boundaries generated from the average of 111 seasons.

In order to verify results, the duration curve of the 2011–1012 heating season was placed in the confidence interval created.

7. Conclusions

We presented in our paper the methodology for the risk-based dimensioning of domestic hot water (DHW) and heating demands. In respect of consumer groups of a discretionary number of apartments, the quantity and intensity of consumption at any time of day and for any period present random and unpredictable fluctuations, therefore they can be considered as stochastic variables and can be described by probability function relations. In case of consumption for any period of time, consumption is characterized by its density and distribution functions. The distribution function is generally normal, determined by two of its parameters, namely the expected value and standard deviation, which are to be specified by measurements. The expected value and standard deviation of DHW consumption of a required period can be determined by means of mathematical statistics based on measurements. It is expedient to carry out measurements for various types of buildings and a variety of numbers of apartments, and to correlate the figures of expected value and standard deviation to an apartment unit. We presented the 99% reliability level sorted duration curve function of consumption intensities.

In building engineering equipment design and operation, it is very important to be aware of developments in meteorological factors, primarily external air temperature. The study has established a probability theory basis for describing external daily mean temperatures and has demonstrated that its probability distribution is of normal distribution. Distribution momentums, the expected value and standard deviation can be determined from meteorological statistics available for various geographical regions. Our study has also presented a method for determining momentum errors and for specifying the confidence range. Mathematical descriptions of the duration curve of external temperatures and degree hours have also been examined. These are also of normal distribution. The area below the duration curve of degree hours is proportionate to heat consumption for heating. The area below the duration curve can be determined by integrating the normal distribution function using the formulas presented. We hope that this paper contributes to the clarification of the problem of the probability theory description of domestic hot water and heating demands.

References

[1] L. Garbai, B. Némethi, Outlines of a stochastic school in building engineering (in Hungarian), Part 1, Hungarian HVAC Magazine XLIX (4) (2000) 3–5.
[2] L. Garbai, District heating (in Hungarian), 2012, ISBN 978-963-279-739-7.
[3] A.R. Thomas, L.J. Brian, Normal Probability Plots and Test for Normality, Statistics Department, The Pennsylvania State University, 1976 http://www.minitab.com/en-us/404.aspx?aspxerrorpath=/uploadedFiles/Shared_Resources/Documents/Articles/normal_probability_plots.pdf
[4] Z. Szánthó, B. Némethi, Measurement Study on Demand of Domestic Hot Water in Residential Buildings, in: Proceedings of the 2nd IASME/WSEAS International Conference on Energy & Environment, Portorose, Slovenia, 15–17 May, 2007, pp. 68–73.
[5] DIN 4708-2, Zentrale Wassererwärmungsanlagen; Regeln zur Ermittlung des Wärmebedarfs zur Erwärmung von Trinkwasser in Wohngebäuden, 1994.
[6] Chen Xi, Lu LIn, Yang Hongxing, Long term operation of solar assisted ground coupled heat pump system for space heating and domestic hot water, Energy and Buildings 43 (2011) 1835–1844.
[7] J.C. Evarts, L.G. Swan, Domestic hot water consumption estimates for solar thermal system sizing, Energy and Buildings 58 (2013) 58–65.
[8] U. Jordan, K. Vajen, Influence of the DHW load profile on the fractional energy savings: a case study of a solar combi-system with TRNSYS simulations, Solar Energy 69 (2001) 197–208.
[9] U. Jordan, K. Vajen, DHWcalc: program to generate domestic hot water profiles with statistical means for user defined conditions, in: Proceedings of the ISES Solar World Congress, Orlando, USA, 8–12 August, 2005.
[10] R. Hendron, J. Burch, G. Barker, Tool for generating realistic residential hot water event schedules, in: Proceedings of the Fourth National Conference of IBPSA-USA, Simbuild 2010, New York City, New York, 2010.
[11] B. Bøhm, Production and distribution of domestic hot water in selected Danish apartment buildings and institutions, Energy Conversion and Management (67) (2013) 152–159.
[12] R. Spur, D. Fiala, D. Nevrala, D. Probert, Influence of the domestic hot-water daily draw-off profile on the performance of a hot-water store, Applied Energy 83 (2006) 749–773.
[13] J. Devore, N. Farnum, Applied Statistic for Engineers and Scientists, Duxbury Press, Pacific Grove, 1999, pp. 656.